

Show $(\text{Im}(E), \text{Im}(H))^T$ is a fixed point of Π

November 1, 2023

Suppose $(E, H)^T$ is the solutions of the non-dimensionalized equation

$$i\omega E = \nabla \times H - J, \quad (1)$$

$$i\omega H = -\nabla \times E. \quad (2)$$

Decompose E and H as $E = \text{Re}(E) + i\text{Im}(E)$ and $H = \text{Re}(H) + i\text{Im}(H)$, we will obtain

$$-\omega \text{Im}(E) = \nabla \times \text{Re}(H) - \text{Re}(J), \quad (3)$$

$$\omega \text{Re}(E) = \nabla \times \text{Im}(H) - \text{Im}(J), \quad (4)$$

$$-\omega \text{Im}(H) = -\nabla \times \text{Re}(E), \quad (5)$$

$$\omega \text{Re}(H) = -\nabla \times \text{Im}(E). \quad (6)$$

Utilizing this relation, one can verify that

$$\tilde{E}(t) = \text{Im}(E) \cos(\omega t) + \text{Re}(E) \sin(\omega t), \quad (7)$$

$$\tilde{H}(t) = \text{Im}(H) \cos(\omega t) + \text{Re}(H) \sin(\omega t). \quad (8)$$

is a solution of

$$\partial_t \tilde{E} = \nabla \times \tilde{H} - \sin(\omega t) \text{Re}(J) - \cos(\omega t) \text{Im}(J), \quad (9)$$

$$\partial_t \tilde{H} = -\nabla \times \tilde{E}. \quad (10)$$

Substitute $(\tilde{E}_0, \tilde{H}_0)^T = (\text{Im}(E), \text{Im}(H))^T$ and $(\tilde{E}(t), \tilde{H}(t))^T$ into the definition of filtering operator Π , we have

$$\begin{aligned} \Pi \begin{pmatrix} \text{Im}(E) \\ \text{Im}(H) \end{pmatrix} &= \Pi \begin{pmatrix} \tilde{E}_0 \\ \tilde{H}_0 \end{pmatrix} = \frac{2}{T} \int_0^T (\cos(\omega t) - \frac{1}{4}) \begin{pmatrix} \tilde{E}(t) \\ \tilde{H}(t) \end{pmatrix} dt \\ &= \frac{\omega}{\pi} \int_0^{2\pi/\omega} (\cos(\omega t) - \frac{1}{4}) \begin{pmatrix} \text{Im}(E) \cos(\omega t) + \text{Re}(E) \sin(\omega t) \\ \text{Im}(H) \cos(\omega t) + \text{Re}(H) \sin(\omega t) \end{pmatrix} dt \\ &= \begin{pmatrix} \text{Im}(E) \\ \text{Im}(H) \end{pmatrix}. \end{aligned} \quad (11)$$